Exercice 14

$$B = \underset{\mathcal{B}}{\mathrm{Mat}}(b) = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} \in \mathcal{M}_{n,1}(\mathbb{R})$$

$\mathbb{H} = \mathrm{Vect}(b)^{\perp}$

Comme $\mathbb{R}^n$ est de dimension finie : $\mathbb{H}^{\perp} = (\mathrm{Vect}(b)^{\perp})^{\perp} = \mathrm{Vect}(b)$

Comme $\|b\| = 1$, b est une base orthonormée de $\mathbb{H}^{\perp}$.

Alors pour $x \in \mathbb{R}^n$: $p_{\mathbb{H}^{\perp}}(x) = \langle x, b \rangle b$

donc $p_{\mathbb{H}}(x) = x - \langle x, b \rangle b$

Donc $\forall (x_1, \dots, x_n) \in \mathbb{R}^n$,

$$p_{\mathbb{H}}(x_1, \dots, x_n) = (x_1, \dots, x_n) - \left(\sum_{k=1}^{n} x_k b_k\right) \cdot (b_1, \dots, b_n)$$

$$= \left(x_1 - \sum_{k=1}^{n} x_k b_k b_1, \dots, x_j - \sum_{k=1}^{n} x_k b_k b_j, \dots, x_n - \sum_{k=1}^{n} x_k b_k b_n\right)$$

Donc :

$$\underset{\mathcal{B}}{\mathrm{Mat}}(p_{\mathbb{H}}) = \begin{pmatrix} 1 - b_1^2 & -b_2 b_1 & -b_3 b_1 & \cdots & -b_n b_1 \\ -b_1 b_2 & 1 - b_2^2 & -b_3 b_2 & \cdots & -b_n b_2 \\ -b_1 b_3 & -b_2 b_3 & 1 - b_3^2 & \cdots & -b_n b_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_1 b_n & -b_2 b_n & -b_3 b_n & \cdots & 1 - b_n^2 \end{pmatrix}$$

$$= \boxed{I_n - B \times {}^tB}$$