On suppose que $X \hookrightarrow \mathcal{U}(\llbracket 1,n \rrbracket)$

ie que $X(\Omega) = \llbracket 1,n \rrbracket$ et $\forall k \in \llbracket 1,n \rrbracket$, $\mathbb{P}(X=k) = \frac{1}{n}$

On pose $Y = \frac{X}{n}$.

* Alors $Y(\Omega) = \left\{\frac{1}{n}; \frac{2}{n}; \dots; \frac{n}{n}\right\}$

* Et $\forall k \in \llbracket 1,n \rrbracket$, $\left(Y = \frac{k}{n}\right) = \left(\frac{X}{n} = \frac{k}{n}\right) = (X=k)$

donc $\mathbb{P}\left(Y = \frac{k}{n}\right) = \frac{1}{n}$

Donc $\boxed{Y \hookrightarrow \mathcal{U}\left(\left\{\frac{1}{n}; \frac{2}{n}; \dots; \frac{n}{n}\right\}\right)}$

* Par linéarité de l'espérance

on a $E(Y) = E\left(\frac{X}{n}\right) = \frac{1}{n} \cdot E(X) = \frac{1}{n} \times \frac{n+1}{2} = \boxed{\frac{n+1}{2n}}$

* $V(Y) = V\left(\frac{X}{n}\right) = \frac{1}{n^2} \cdot V(X) = \frac{1}{n^2} \times \frac{n^2-1}{12} = \boxed{\frac{n^2-1}{12n^2}}$