TD 2

On pose $S_n = \sum_{1 \leq j < i \leq n} ij$

Alors $S_n = \sum_{i=2}^{n} \left(\sum_{j=1}^{i-1} ij \right)$

Pour $i \in [\![\ , n]\!]$: $\sum_{j=1}^{i-1} ij = i \times \sum_{j=1}^{i-1} j = i \frac{(i-1)i}{2} = \frac{i^3 - i^2}{2}$

donc $S_n = \sum_{i=2}^{n} \frac{i^3 - i^2}{2} = \frac{1}{2} \sum_{i=2}^{n} i^3 - \frac{1}{2} \sum_{i=2}^{n} i^2$

$$= \frac{1}{2} \times \left[\frac{n^2(n+1)^2}{4} - 1^3 \right] - \frac{1}{2} \left[\frac{n(n+1)(2n+1)}{6} - 1^2 \right]$$

$$= \frac{n(n+1)}{24} \times \left[3n(n+1) - 2(2n+1) \right]$$

$$= \frac{n(n+1)}{24} \times \left[3n^2 - n - 2 \right] = \frac{n(n+1)}{24} \times (3n+2)(n-1)$$

$$= \boxed{\frac{n(n^2-1)(3n+2)}{24}}$$

<u>vérification</u> si $n = 2$ alors $S_n = 2 \times 1 = 2$

et $\frac{n(n^2-1)(3n+2)}{24} = \frac{2 \times 3 \times 8}{24} = 2$ ok.