TD3

① On a : $2\cos\left(2x+\frac{\pi}{3}\right)=\sqrt{3} \iff \cos\left(2x+\frac{\pi}{3}\right)=\cos\frac{\pi}{6}$

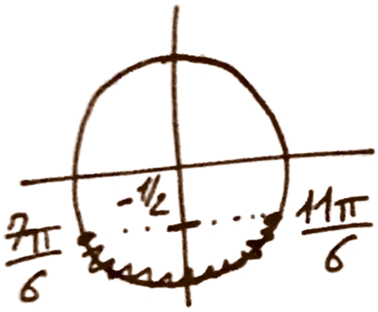
$$\iff 2x+\frac{\pi}{3}=\pm\frac{\pi}{6}\ [2\pi]$$

$$\iff \begin{cases} x=-\frac{\pi}{12}\ [\pi] \\ x=-\frac{\pi}{2}\ [\pi] \end{cases}$$

L'ensemble des solutions est :

$$\mathcal{S}=\left\{-\frac{\pi}{12}+k\pi ; k\in\mathbb{Z}\right\}\cup\left\{-\frac{\pi}{2}+k\pi ; k\in\mathbb{Z}\right\}$$

② Pour $x\in[0,2\pi[$: $\sin(x)\leq-\frac{1}{2} \iff x\in\left[\frac{7\pi}{6};\frac{11\pi}{6}\right]$



Dans $\mathbb{R}$, l'ensemble des solutions est :

$$\mathcal{S}=\bigcup_{k\in\mathbb{Z}}\left[\frac{7\pi}{6}+2k\pi ; \frac{11\pi}{6}+2k\pi\right]$$

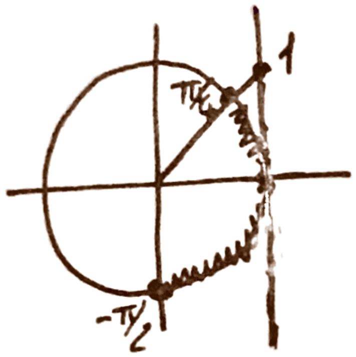
③ Pour $t\in]-\pi,\pi]$: $\cos t\geq 0 \iff t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Pour $t\in\mathbb{R}$: $\cos t\geq 0 \iff t\in\bigcup_{k\in\mathbb{Z}}\left[-\frac{\pi}{2}+2k\pi ; \frac{\pi}{2}+2k\pi\right]$

Donc pour $x\in\mathbb{R}$: $\cos(2x)\geq 0 \iff 2x\in\bigcup_{k\in\mathbb{Z}}\left[-\frac{\pi}{2}+2k\pi ; \frac{\pi}{2}+2k\pi\right]$

$$\iff x\in\bigcup_{k\in\mathbb{Z}}\left[-\frac{\pi}{4}+k\pi ; \frac{\pi}{4}+k\pi\right]$$

④ Pour $x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$: $\tan x \leq 1 \iff x \in \left]-\frac{\pi}{2}, \frac{\pi}{4}\right]$



Pour $x \in D_{\tan}$: $\tan x \leq 1 \iff x \in \bigcup_{k \in \mathbb{Z}} \left]-\frac{\pi}{2}+k\pi, \frac{\pi}{4}+k\pi\right]$

⑤ Pour $t \in D_{\tan}$: $\tan t > -1 \iff t \in \bigcup_{k \in \mathbb{Z}} \left]-\frac{\pi}{4}+k\pi, \frac{\pi}{2}+k\pi\right[$

Pour $x+\frac{\pi}{4} \in D_{\tan}$ ie $x \neq \frac{\pi}{4}\ [\pi]$

$\tan\left(x+\frac{\pi}{4}\right) > -1 \iff x \in \bigcup_{k \in \mathbb{Z}} \left]-\frac{\pi}{2}+k\pi, \frac{\pi}{4}+k\pi\right[$

⑥ $2t^2+3t+1=0 \iff t=-1$ ou $-\frac{1}{2}$

Donc $2\cos^2 x+3\cos x+1=0 \iff \cos x=-1$ ou $\cos x=-\frac{1}{2}$

$\iff \cos x = \cos \pi$ ou $\cos x = \cos \frac{2\pi}{3}$

$\iff x=\pi\ [2\pi]$ ou $x=-\pi\ [2\pi]$ ou $x=\frac{2\pi}{3}\ [2\pi]$

ou $x=-\frac{2\pi}{3}\ [2\pi]$

$\iff x=\pi\ [2\pi]$ ou $x=\pm\frac{2\pi}{3}\ [2\pi]$

⑦ $\sin^2 x + 3\cos x - 1 < 0$

$\iff 1 - \cos^2 x + 3\cos x - 1 < 0$

$\iff \cos x \times (3 - \cos x) < 0$

$\iff \cos x < 0$ car $3 - \cos x > 0$

$\iff x \in \bigcup_{k \in \mathbb{Z}} \left]\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi\right[$

⑧ On a $\forall x \in \mathbb{R}$, $\cos(2x) - \sqrt{3}\sin(2x) = 2 \times \left(\frac{1}{2}\cos(2x) - \frac{\sqrt{3}}{2}\sin(2x)\right)$

$= 2 \times \cos\left(x + \frac{\pi}{3}\right)$

Donc $\cos(2x) - \sqrt{3}\sin(2x) = 1$

$\iff \cos\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

$\iff x + \frac{\pi}{3} = \pm\frac{\pi}{3}\ [2\pi]$

$\iff x = 0\ [2\pi]$ ou $x = -\frac{2\pi}{3}\ [2\pi]$

⑨ $\sin^2\left(2x + \frac{\pi}{6}\right) = \cos^2\left(x + \frac{\pi}{3}\right)$

$\iff \cos^2\left(\frac{\pi}{2} - 2x - \frac{\pi}{6}\right) = \cos^2\left(x + \frac{\pi}{3}\right)$ car $\sin t = \cos\left(\frac{\pi}{2} - t\right)$

$\iff \cos^2\left(\frac{\pi}{3} - 2x\right) = \cos^2\left(x + \frac{\pi}{3}\right)$

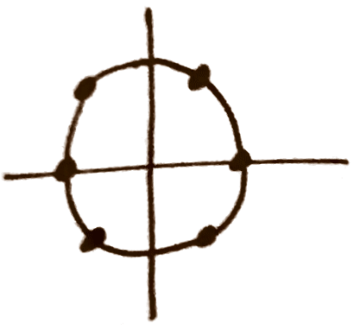
$\iff \cos\left(\frac{\pi}{3} - 2x\right) = \cos\left(x + \frac{\pi}{3}\right)$ ou $\cos\left(\frac{\pi}{3} - 2x\right) = -\cos\left(x + \frac{\pi}{3}\right)$

$\iff \cos\left(\frac{\pi}{3} - 2x\right) = \cos\left(x + \frac{\pi}{3}\right)$ ou $\cos\left(\frac{\pi}{3} - 2x\right) = \cos\left(x + \frac{\pi}{3} + \pi\right)$

$\iff \frac{\pi}{3} - 2x = \pm\left(x + \frac{\pi}{3}\right) [2\pi]$ ou $\frac{\pi}{3} - 2x = \pm\left(x + \frac{4\pi}{3}\right)$

$\iff x = 0 \left[\frac{2\pi}{3}\right]$ ou $x = \frac{2\pi}{3} [2\pi]$ ou $x = -\frac{\pi}{3} \left[\frac{2\pi}{3}\right]$

ou $x = \frac{5\pi}{3} [2\pi]$



$\iff x = 0 \left[\frac{\pi}{3}\right]$