# Exercice 6 du TD 3



(2) $\cos^2 x \times \sin^4 x = \left(\dfrac{e^{ix}+e^{-ix}}{2}\right)^2 \times \left(\dfrac{e^{ix}-e^{-ix}}{2i}\right)^4$

$= \dfrac{1}{2^6 . i^4} \times \left(e^{i2x}+2+e^{-i2x}\right) \times \left(e^{i4x}-4e^{i2x}+6-4e^{-i2x}+e^{-i4x}\right)$

$= \dfrac{1}{2^6} \times \left(e^{i6x}-2e^{i4x}-e^{i2x}+4-e^{-i2x}-2e^{-i4x}+e^{-i6x}\right)$

$= \dfrac{1}{32} \times \left(\cos(6x)-2\cos(4x)-\cos(2x)+2\right)$

(3) $\sin^5(x) = \left(\dfrac{e^{ix}-e^{-ix}}{2i}\right)^5 = \dfrac{1}{2^5 i}\left(e^{ix}-e^{-ix}\right)^5$

$= \dfrac{1}{2^5 . i}\left(e^{i5x}-5e^{i3x}+10e^{ix}-10e^{-ix}+5e^{-i3x}-e^{-i5x}\right)$

$= \dfrac{1}{16}\left(\sin(5x)-5\sin(3x)+10\sin(x)\right)$

(4) $\cos^3(2x) \times \sin^3(x) = \left(\dfrac{e^{i2x}+e^{-i2x}}{2}\right)^3 \times \left(\dfrac{e^{ix}-e^{-ix}}{2i}\right)^3$

$= \dfrac{1}{2^6 . i^3} \times \left(e^{i6x}+3e^{i2x}+3e^{-i2x}+e^{-i6x}\right) \times \left(e^{i3x}-3e^{ix}+3e^{-ix}-e^{-i3x}\right)$

$= \dfrac{1}{-2^6 i}\left(e^{i9x}-3e^{i7x}+6e^{i5x}-10e^{i3x}+12e^{ix}-12e^{-ix}+10e^{-i3x}-6e^{-i5x}+3e^{-i7x}-e^{-i9x}\right)$

$= \dfrac{1}{32}\left(-\sin(9x)+3\sin(7x)-6\sin(5x)+10\sin(3x)-12\sin(x)\right)$

5 $\cos(2x) \times \cos^3(x) = \dfrac{e^{i2x}+e^{-i2x}}{2} \times \left(\dfrac{e^{ix}+e^{-ix}}{2}\right)^3$

$= \dfrac{1}{2^4}\left(e^{i2x}+e^{-i2x}\right) \times \left(e^{i3x}+3e^{ix}+3e^{-ix}+e^{-i3x}\right)$

$= \dfrac{1}{2^4}\left(e^{i5x}+3e^{i3x}+4e^{ix}+4e^{-ix}+3e^{-i3x}+e^{-i5x}\right)$

$= \dfrac{1}{8}\left(\cos(5x)+3\cos(3x)+4\cos(x)\right)$

1 $\cos(x)^6 = \dfrac{1}{2^6}\left(e^{ix}+e^{-ix}\right)^6$

$= \dfrac{1}{2^6}\left(e^{i6x}+6e^{i4x}+15e^{i2x}+20+15e^{-i2x}+6e^{-i4x}+e^{-i6x}\right)$

$= \dfrac{1}{2^6}\left(2\cos(6x)+12\cos(4x)+30\cos(2x)+20\right)$

$= \dfrac{1}{32}\left(\cos(6x)+6\cos(4x)+15\cos(2x)+10\right)$