Ex 7 TD7

1. $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{cases} \boxed{x_1} + x_2 + x_3 = y_1 \\ \boxed{x_2} + x_3 = y_2 \\ \boxed{x_3} = y_3 \end{cases} \iff \begin{cases} x_1 = y_1 - y_2 \\ x_2 = y_2 - y_3 \\ x_3 = y_3 \end{cases}$$

donc A est inversible et $\boxed{A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}$

2. $B = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 3 \end{pmatrix}$

$$\begin{cases} \boxed{x_1} + x_2 + 2x_3 = y_1 \\ x_1 + 2x_2 + 3x_3 = y_2 \\ -x_2 + 3x_3 = y_3 \end{cases} \iff \begin{cases} \boxed{x_1} + x_2 + 2x_3 = y_1 \\ \boxed{x_2} + x_3 = y_2 - y_1 \quad L_2 \leftarrow L_2 - L_1 \\ -x_2 + 3x_3 = y_3 \end{cases}$$

$$\iff \begin{cases} \boxed{x_1} + x_2 + 2x_3 = y_1 \\ \boxed{x_2} + x_3 = y_2 - y_1 \\ \boxed{4x_3} = y_3 + y_2 - y_1 \quad L_3 \leftarrow L_3 + L_2 \end{cases}$$

$$\iff \begin{cases} x_1 = \frac{1}{4}(9y_1 - 5y_2 - y_3) \\ x_2 = \frac{1}{4}(-3y_1 + 3y_2 - y_3) \\ x_3 = \frac{1}{4}(-y_1 + y_2 + y_3) \end{cases}$$

donc A est inversible et $\boxed{A^{-1} = \frac{1}{4}\begin{pmatrix} 9 & -5 & -1 \\ -3 & 3 & -1 \\ -1 & 1 & 1 \end{pmatrix}}$

3. $C = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & -2 & 0 \\ 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & -3 \end{pmatrix}$

$$\begin{cases} x_1 + x_2 + 2x_4 = y_1 \\ -2x_3 = y_2 \\ x_1 + 2x_2 + 3x_4 = y_3 \\ x_2 - 3x_4 = y_4 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x_1 + x_2 - 2x_4 = y_1 \\ -2x_3 = y_2 \\ x_2 + x_4 = y_3 - y_1 \quad L_3 \leftarrow L_3 - L_1 \\ x_2 - 3x_4 = y_4 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x_1 + x_2 - 2x_4 = y_1 \\ x_2 + x_4 = y_3 - y_1 \quad L_2 \leftrightarrow L_3 \\ -2x_3 = y_2 \\ x_2 - 3x_4 = y_4 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x_1 + x_2 - 2x_4 = y_1 \\ x_2 + x_4 = y_3 - y_1 \\ -2x_3 = y_2 \\ -4x_4 = y_4 - y_3 + y_1 \quad L_4 \leftarrow L_4 - L_2 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x_1 = \frac{1}{4}(2y_1 - 5y_3 + y_4) \\ x_2 = \frac{1}{4}(-3y_1 + 3y_3 + y_4) \\ x_3 = \frac{1}{4}(-2y_2) \\ x_4 = \frac{1}{4}(-y_1 + y_3 - y_4) \end{cases}$$

donc A est inversible et $\boxed{A^{-1} = \frac{1}{4}\begin{pmatrix} 2 & 0 & -5 & 1 \\ -3 & 0 & 3 & 1 \\ 0 & -2 & 0 & 0 \\ -1 & 0 & 1 & -1 \end{pmatrix}}$